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## Teaneck Public Schools

Summer Assignment

## TEAMS 10 - Algebra II Trig Honors



## Below are the directions and policies for the summer packet

- This is a summer packet for students entering into Algebra II/H Algebra II with Trig course. This packet is due on the first day of school.
- The summer assignment is due the first day of class. On the first day of class, teachers will collect summer assignments. Any student who does not have the assignment will be given one by the teacher. Late projects will lose 10 points each day.
- This summer assignment will be graded on completion. Completion is defined as having all work shown in the space provided to receive full credit, and a parent/guardian signature. A preliminary assessment (administered during the first week of school) will be based on the topics covered in this packet.
- Any student who registers as a new attendee of Teaneck High School after August $15^{\text {th }}$ will have an extra week to complete the summer assignment.
- Summer assignment packets will be available on the district web site and available in the THS guidance office.

The list below contains topics covered in Algebra I that students are expected to know upon entering Algebra II. Each section contains 3 steps: Step 1 allows students to review the mathematical concept with Khan Academy and/or Desmos, Step 2 shows how the concepts are used in problems with detailed solutions, and Step 3 provide opportunities for students to try the problems on their own.

Check $(\checkmark)$ as you complete each section:

## Part 1: Solving Systems of Linear Equations

___1.1. Solving Systems of Linear Equation by Graphing
1.2. Solving Systems of Linear Equations by Substitution
1.3. Solving Systems of Linear Equations by Elimination
1.4. NJSLA Questions

## Part 2: Solving Quadratic Equation

___ 2.1. Solving Quadratic Equations by Graphing
2.2. Solving Quadratic Equations by Using Square Roots
2.3. Solving Quadratic Equations by Completing the Square
2.4. Solving Quadratic Equations by Using the Quadratic Formula

Part 3: Putting it all together...
3.1. Solving Nonlinear Systems of Equations
3.2. NJSLA Questions

## Part 1: Solving Systems of Linear Equations

### 1.1. Solving Systems of Linear Equation by Graphing

Step 1: Watch this video: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-graphically/v/solving-linear-systems-by-graphing

Then, go to student.desmos.com and type in Q N J U W Z and complete the task.

Step 2: Review the solved examples
Example 1) Solve the system of linear equations by graphing:

$$
y=-x+5 \quad y=2 x-1
$$

$$
\begin{array}{ll}
\text { Equation 1 } & \text { Equation 2 } \\
y=-x+5 & y=2 x-1 \\
3=-2+5 & 3=2(2)-1 \\
3=3 \checkmark & 3=4-1 \\
& 3=3 \checkmark
\end{array}
$$

The solution is $(2,3)$.


Example 2) Solve the system of linear equations by graphing:

$$
2 x+y=5 \quad 3 x-2 y=4
$$

$$
\begin{aligned}
2 x+y & =5 \\
2 x-2 x+y & =5-2 x \\
y & =-2 x+5
\end{aligned}
$$



Check Equation 1

$$
\begin{aligned}
2 x+y & =5 \\
2(2)+1 & \stackrel{?}{=} 5 \\
4+1 & \stackrel{?}{=} 5 \\
5 & =5
\end{aligned}
$$

Equation 2

$$
3 x-2 y=4
$$

$$
3(2)-2(1) \stackrel{?}{=} 4
$$

$$
6-2 \stackrel{?}{=} 4
$$

$$
4=4 \checkmark
$$

The solution is $(2,1)$.

Step 3: Solve the system of equations by graphing.

1. $3 x-5 y=2$
$y=2$

2. $y=-\frac{3}{2} x-3$

$$
y=\frac{1}{2} x+5
$$


2. $-x+4 y=-10$
$2 x-3 y=5$

4. $3 x+3 y=-3$
$5 x+2 y=1$

5. For each rectangle below, write a linear equation that represents the area $y$ of the rectangle. Solve this system of two linear equations by graphing. Interpret your solution.



### 1.2. Solving Systems of Linear Equations by Substitution

Step 1: Watch the video: https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-systems-topic/cc-8th-systems-with-substitution/v/the-substitution-method

Step 2: Review the solved examples
Example 1) Solve the following equations by substitution

$$
\begin{array}{ll}
y=-x+3 & \text { Equation } \\
3 y+5 x=-1 & \text { Equations }
\end{array}
$$

Substitute $-x+3$ for $y$ in Equation 2 ad solve for $x$.

$$
\begin{aligned}
3 y+5 x & =-1 \\
3(-x+3)+5 x & =-1 \\
-3 x+9+5 x & =-1 \\
2 x+9 & =-1 \\
2 x & =-10 \\
x & =-5
\end{aligned}
$$

Substitute -5 for $x$ in Equation l and solve for $y$.

$$
\begin{aligned}
& y=-x+3 \\
& y=-(-5)+3 \\
& y=5+3 \\
& y=8
\end{aligned}
$$

The solution is $(-5,8)$.

Example 2) Solve the system of equations by substitution

$$
2 x-y=2 \quad 2 x+y=6
$$



Step 3: Solve the systems of equations by substitution.

4. Write an equation that represents the sum of the angle measures of the parallelogram and use your equations and the equation shown to find the values of $x$ and $y$.


Step 1. Watch the video: https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-equivalent-systems-of-equations/v/solving-systems-of-equations-by-elimination

Step 2. Review the solved examples.


| Solve the system: $\left.\begin{array}{l}5 x-3 y=-1 \\ 3 x+2 y=7\end{array}\right\}$ |
| :---: |
| $\begin{aligned} & \begin{array}{l} 5 x-3 y=-1 \\ 3 x+2 y=7 \end{array}\left\{\begin{array}{rr} 10 x-6 y & =-2 \\ x(3) & \frac{\oplus x+6 y}{}=21 \end{array}\right\} \\ & 19 x=19 \\ & \text { BACK SuBst. } \end{aligned}$ $\begin{aligned} 3(1)+2 y & =7 \\ 2 y & =4 \\ y & =2 \end{aligned} \quad \text { Ans: }(1,2)$ |

Step 3. Solve the systems of equations by elimination.

| 1 | $\begin{aligned} & 4 x-y=5 \\ & 3 x+y=9 \end{aligned}$ | 2. | $\begin{aligned} & 2 x-7 y=5 \\ & x-y=10 \end{aligned}$ |
| :---: | :---: | :---: | :---: |
|  | $-10 x+3 y=-30$ |  | $7 x-6 y=9$ |
| 3. | $15 x-8 y=45$ | 4. | $5 x+2 y=19$ |

5. You are ordering T-shirts for Teaneck HS. The table shows the order for 45 students.
a. How many students ordered medium and large shirts?

| Small | Medium | Large |
| :---: | :---: | :---: |
| 11 | $x$ | $y$ |

b. The number of students who ordered a medium T-shirt was two less than the number of students who ordered a large T-shirt. Write a system of linear equations that represents the number of students who ordered medium and large T-shirts.
c. Solve the system of linear equations by elimination.
d. You are ordering 10 additional medium and large T-shirts for new members who might join the school.

Based on your answers in part c, how many of each size would you order? Explain.

### 1.4. NJSLA Questions

Make sure to show all work!

## 10.

A family compares the costs of renting a truck from two different companies for its 2-day move to another state. The costs are shown in the table.

Truck Rental Costs

| Item | Company X | Company Y |
| :--- | :---: | :---: |
| base rental <br> charge | \$29.95 per <br> day | $\$ 19.95$ per <br> day |
| mileage <br> charge | 59 cents <br> per mile | 79 cents <br> per mile |
| drop-off <br> charge | $\$ 150$ | included |
| insurance | $\$ 18$ per day | $\$ 26$ per day |

## Part A

Create a model that can be used to determine the rental cost of each truck for the 2-day move. Describe the process you used to determine your model.

Use your model to determine the number of miles when the rental costs of the two trucks will be equal.

Answer:

## 10. (continued from previous page)

## Part B

The family estimates they will travel 750 miles total. In addition to the truck rental cost, they will also need to pay for gasoline. The price of gasoline is $\$ 3.50$ per gallon across the states they will be traveling. The truck from Company X averages 10 miles per gallon; the truck from Company Y averages 7 miles per gallon.

Which of the two trucks should the family rent? Provide an answer supported by valid mathematical reasoning and/or calculations.

Answer:

## Part 2: Solving Quadratic Equation

### 2.1. Solving Quadratic Equations by Graphing

Step 1. Watch the video: https://www.khanacademy.org/math/algebra2/advanced-functions/systems-of-quadratic-equations/v/non-linear-systems-of-equations-1

Then, go to student.desmos.com and type in XN 6 X 5 A and complete the task.

Step 2. Review the solved examples.
Solve $x^{2}+4 x=-4$ by graphing.

$$
\begin{aligned}
x^{2}+4 x & =-4 \\
x^{2}+4 x+4 & =0
\end{aligned}
$$

The only $x$-intercept is at the vertex, $(-2,0)$.

$$
\text { So, the solution is } x=-2 \text {. }
$$



Solve $x^{2}+2 x=8$ by graphing.

$$
\begin{array}{r}
x^{2}+2 x=8 \\
x^{2}+2 x-8=0
\end{array}
$$

The $x$-intercepts are -4 and 2 .

$$
\begin{aligned}
& \text { So, the solutions are } \\
& x=-4 \text { and } x=2 \text {. }
\end{aligned}
$$



Step 3. Solve the quadratic equations by graphing. Feel free to utilize Desmos calculator!

5. The height $h$ (in feet) of a ball in a baseball game can be modeled by $h=-16 t^{2}+28 t+8$, where $t$ is the time (in seconds).
a. Do both $t$-intercepts of the graph of the function have meaning in this situation? Explain.
b. No one caught the ball. After how many seconds did the ball hit the ground?


### 2.2. Solving Quadratic Equations Using Square Roots

Step 1. Watch the video: https://www.khanacademy.org/math/algebra/quadratics/quadratics-square-root/v/simple-quadratic-equation

Step 2. Review the solved examples.
a. Solve $2 x^{2}-32=0$ using square roots.

$$
\begin{aligned}
2 x^{2}-32 & =0 & & \text { Write the equation. } \\
2 x^{2} & =32 & & \text { Add } 32 \text { to each side. } \\
x^{2} & =16 & & \text { Divide each side by } 2 . \\
x & = \pm \sqrt{16} & & \text { Take the square root of each side. } \\
x & = \pm 4 & & \text { Simplify. }
\end{aligned}
$$

The solutions are $x=-4$ and $x=4$.
b. Solve $x^{2}-8=-8$ using square roots.

$$
\begin{array}{rlr}
x^{2}-8 & =-8 & \text { Write the equation. } \\
x^{2} & =0 & \text { Add } 8 \text { to each side. } \\
x & =0 & \text { Take the square root of each side. }
\end{array}
$$

The only solution is $x=0$.
c. Solve $-2 x^{2}+3=27$ using square roots.

$$
\begin{aligned}
-2 x^{2}+3 & =27 & & \text { Write the equation. } \\
-2 x^{2} & =24 & & \text { Subtract } 3 \text { from each side. } \\
x^{2} & =-12 & & \text { Divide each side by }-2 .
\end{aligned}
$$

The square of a real number cannot be negative. So, the equation has no real solutions.

Solve $(x-3)^{2}=16$ using square roots.

$$
\begin{aligned}
(x-3)^{2} & =16 & & \text { Write the equation. } \\
x-3 & = \pm \sqrt{16} & & \text { Take the square root of each side. } \\
x-3 & = \pm 4 & & \text { Simplify. } \\
x & =3 \pm 4 & & \text { Add } 3 \text { to each side. }
\end{aligned}
$$

$$
\text { So, the solutions are } x=3-4=-1 \text { and } x=3+4=7 \text {. }
$$

Step 3. Solve the equations using square roots.

| 1. $x^{2}-8=10$ | 2. $-x^{2}+25=25$ |
| :--- | :--- |
| 3. $(x+2)^{2}=9$ | $4.2 x^{2}+10=0$ |

Determine the number of real solutions of the equation. Then, solve the equation using square roots.
5. $x^{2}=36$
6. $x^{2}=-16$
7. $x^{2}=0$
8. A can of juice has a height of 10 inches and a volume of $160 \pi$ cubic inches. The volume of a can with radius $r$ is given by the formula $V=\pi r^{2} h$.
a. Write an equation describing this situation, where $r$ is the radius of the can.
b. Find the radius of the can.

### 2.3. Solving Quadratic Equations by Completing the Square

Step 1. Watch the video: https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/v/solving-quadratic-equations-by-completing-the-square

Step 2. Review the solved examples: https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-by-completing-the-square/e/completing the square in quadratic expressions

Solve $x^{2}-18 x=-17$ by completing the square.

$$
\begin{aligned}
x^{2}-\frac{18 x}{v} & =-17 \quad \frac{b}{2}=\frac{-18}{2}=-9 \\
x^{2}-18 x+(-9)^{2} & =-17+(-9)^{2} \\
\sqrt{(x-9)^{2}} & =\sqrt{64} \\
x-9 & = \pm 8 \\
+9 & +9 \\
x & =9 \pm 8
\end{aligned}
$$

The solutions are $x=9+8=17$ and $x=9-8=1$.

Solve $2 x^{2}+12 x-10=0$ by completing the square.

$$
\begin{gathered}
2 x^{2}+12 x-10=0 \\
+\frac{10}{2 x^{2}+12 x}=\frac{+10}{2} \\
x^{2}+\frac{6 x}{2}=5 \quad \frac{6}{2}=\frac{6}{2}=3 \\
x^{2}+6 x+3^{2}=5+3^{2} \\
\sqrt{(x+3)^{2}}=\sqrt{14} \\
x+3= \pm \sqrt{14} \\
\frac{-3}{x}=\frac{-3}{-3} \pm \sqrt{14} \\
\text { The solutions are } x=-3+\sqrt{14} \approx 0.74 \\
\text { and } x=-3-\sqrt{14} \approx-6.74 .
\end{gathered}
$$

Step 3. Solve the equations by completing the square.

| 1. $x^{2}-6 x+18=0$ | 2. $\quad x^{2}+2 x-15=0$ |  |
| :--- | :--- | :--- |
| 3. $-4 x^{2}-16 x+19=-17$ | $4 . \quad 2 x^{2}-16 x+20=0$ |  |
|  |  |  |

7. The product of two consecutive odd integers that are positive is 323 .
a. Write an equation to find the integers.
b. Find the two integers.

### 2.4. Solving Quadratic Equations by Using the Quadratic Formula

Step 1. Watch the video: https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/v/quadratic-formula-1

Step 2. Review the solved examples
The quadratic formula says that
$x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
for any quadratic equation like:
$a x^{2}+b x+c=0$

## Example:

We're given an equation and asked to solve for $q$ :

$$
0=-7 q^{2}+2 q+9
$$

This equation is already in the form $a x^{2}+b x+c=0$, so we can apply the quadratic formula where $a=-7, b=2, c=9$ :

$$
\begin{aligned}
& q=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} \\
& q=\frac{-2 \pm \sqrt{2^{2}-4(-7)(9)}}{2(-7)}
\end{aligned}
$$

## Checking solutions:

$$
q=\frac{-2 \pm \sqrt{256}}{-14}
$$

\[

\]

$$
q=\frac{-2 \pm 16}{-14}
$$

$$
q=\frac{-2+16}{-14}, q=\frac{-2-16}{-14}
$$

$$
q=-1 \quad, q=\frac{9}{7}
$$

Step 3. Solve the equations by using the quadratic formula.

| 1. $x^{2}+5 x+14=0$ | 2. $3 x^{2}-2 x-1=0$ |
| :--- | :--- |
|  |  |
| 3. $-5 x^{2}+9 x=-3$ | $4.3 x^{2}-2 x=-6$ |
| 5. |  |

7. Consider the equation $3 x^{2}+5 x+6=0$.
a. Use the discriminant to determine the number of solutions. (Discriminant review: https://www.khanacademy.org/math/algebra/quadratics/solving-quadratics-using-the-quadratic-formula/a/discriminant-review)
b. Change the sign of $c$ in the equation. Write the new equation: $\qquad$
c. Use the discriminant to determine the number of solutions of the new equation. Did your answer change? Explain.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

### 3.1. Solving Systems of Nonlinear Equations

Step 1: Watch this video: https://www.khanacademy.org/math/algebra2/advanced-functions/systems-of-quadratic-equations/v/non-linear-systems-of-equations-1

Step 2: Review the solved examples
Solve the system by substitution.

$$
\begin{aligned}
& y=x^{2}-4 x \\
& y=-4 \\
& y=x^{2}-4 x \\
& -4=x^{2}-4 x \\
& +\underline{4}+4 \\
& 0=x^{2}-4 x+4 \\
& 0=(x-2)^{2} \\
& 0=x-2 \\
& \frac{+2}{2}=x
\end{aligned}
$$

Solve the system by elimination.

$$
\begin{aligned}
& y=x^{2}+2 x-5 \\
& y=2 x-1
\end{aligned}
$$



$$
\begin{aligned}
y & =2 x-1 \\
Y & =2(-2)-1 \\
& =-4-1 \\
& =-5
\end{aligned}
$$

The solution is $(2,-4)$.
The solutions are $(2,3)$ and $(-2,-5)$.
Solve the system by graphing.


Equation 2

$$
\begin{array}{ll}
y=x-1 & y=x-1 \\
0=1-1 & 5 \stackrel{?}{=}=6-1 \\
0=0- & 5=5
\end{array}
$$

The solutions are $(1,0)$ and $(6,5)$.

Step 3.
\#1-2. Solve each system by graphing.

\#3-6. Solve each system by substitution.
3. $\begin{aligned} y & =x-4 \\ y & =x^{2}-3 x-4\end{aligned}$

$$
y=x^{2}-3 x-4
$$

4. $y=-x^{2}+3$
$y=3 x-7$
5. $y+3=x^{2}$
$y=-3$
\#7-10. Solve each system by elimination.
6. | $y$ | $=-4 x^{2}+8 x-8$ |
| ---: | :--- | :--- |
| $y$ | $=-8 x+4$ | \left\lvert\, \(\begin{array}{ll}8. \quad y=2 x^{2}+x-6 <br>

\& y=x+2\end{array}\right.\)

$$
y=-8 x+4
$$

$$
y=x+2
$$

9. $y=2 x^{2}+2 x$
$y=-2 x+6$

$$
\text { 10. } \begin{aligned}
y & =x^{2}-4 x+7 \\
y & =-x+11
\end{aligned}
$$

11. The graphs of $f(x)=1.6 x^{2}+2 x-0.6$ and $g(x)=-2.5 x^{2}-2 x-4.2$ do not intersect. Change the value(s) of $c$ in one or both functions of $f$ and $g$ until the two graphs do intersect. Write your new system of equations and determine the intersection point(s), rounding to the nearest hundredth, if necessary.

### 3.2. NJSLA Questions

Make sure to show all work!
16. Functions $f$ and $g$ are defined below.

$$
\left\{\begin{array}{l}
f(x)=\frac{1}{2 x} \\
g(x)=x^{2}
\end{array}\right.
$$

The graphs of $y=f(x)$ and $y=g(x)$ intersect at point $P$.
Determine the $x$-coordinate of $P$. Round your answer to the nearest tenth.
18. Let $f(x)=a x^{2}$ where $a>0$, and let $g(x)=m x+b$ where $m>0$ and $b<0$.

The equation $f(x)=g(x)$ has $n$ distinct real solution(s). What are all the possible values of $n$ ? Justify your answers.

Enter your answers and your justification in the space provided.
33.

$$
\left\{\begin{array}{l}
y=1-x^{2} \\
y=2-x
\end{array}\right.
$$

How many points of intersection does the given system of equations have?
A. none
B. one
C. two
D. infinitely many
8.

Solve the quadratic equation $(2 x-3)^{2}=6(3-2 x)$.
Select from the drop-down menus to correctly complete the sentence.

| The solutions are | Choose... | and | Choose... | v |
| :---: | :---: | :---: | :---: | :---: |
|  | Choose. |  | Choose. |  |
|  | -2.25 |  | -2.25 |  |
|  | -1.5 |  | -1.5 |  |
|  | -1.0 |  | -1.0 |  |
|  | 1 |  | 1 |  |
|  | 1.5 |  | 1.5 |  |
|  | 2.25 |  | 2.25 |  |

13. 

The system of equations shown is graphed on the coordinate plane. The graphs of the equations form a line and a parabola that intersect at two points.

$$
\left\{\begin{array}{c}
x+y=5 \\
x^{2}+y=11
\end{array}\right.
$$

One point of intersection is $(3,2)$. What are the coordinates of the other point?
Enter your answers in the boxes.

15.

M40974
At noon, a tank contains 100 gallons of water. The table shows the input and output of water for pipes A, B , and C . The pipes begin operating simultaneously at noon.

| Pipe | A | B | C |
| :--- | :---: | :---: | :---: |
| Flow in (gallons per minute) | $a(x)=25 x$ | $b(x)=10 x$ |  |
| Flow out (gallons per minute) |  |  | $c(x)=30 x$ |

Let $T(x)$ represent the amount of water in the tank $x$ minutes after all of pipes $\mathrm{A}, \mathrm{B}$, and C are opened. Which function represents $T(x)$ ?A. $T(x)=100+a(x)+b(x)+c(x)$B. $T(x)=a(x)+b(x)-c(x)$C. $T(x)=100+a(x)+b(x)-c(x)$D. $T(x)=a(x)+b(x)+c(x)$
16.

Which of the choices listed is a solution to $2 x^{2}+4 x+9=0$ ?A. $x=-1-i \sqrt{14}$B. $x=-1+\sqrt{14}$C. $x=\frac{-2+\sqrt{14}}{2}$D. $x=\frac{-2-i \sqrt{14}}{2}$

